Int J Theor Phys (2011) 50: 48–55
DOI 10.1007/s10773-010-0492-1

Electromagnetic Quasinormal Modes in Hořava-Lifshitz Gravity

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Received: 15 July 2010 / Accepted: 7 September 2010 / Published online: 15 September 2010

Abstract The electromagnetic quasinormal modes of Hořava-Lifshitz black hole is investigated by means of six-order WKB approach. We in this paper compare the quasinormal modes of this black hole with the charged black hole's cases (we here take a regular charged black hole and Reissner-Nordström black hole for example). The numerical results of Hořava-Lifshitz's quasinormal modes frequency show that the absolute value of imaginary part decrease as the parameter α increase. The fact means that charge in this spacetime make the quasinormal modes damp at a slower rate.

Keywords Quasinormal modes · Hořava-Lifshitz black hole · Charged black hole · Electromagnetic field · Six-order WKB approach

1 Introduction

In General Relativity, it is predicted that black hole will arise from the compact stars with huge mass. Near the horizon of black hole, the gravity is so big that any matter will inevitably fall into the black hole. People have prove that the property of black hole is very simple, since No hair theorem of black hole shows that black hole can be determined by mass, charge and angular momentum, and the research about black hole is similar to the study about hydrogen atom in quantum theory. Therefore, many theoretical physicist is interesting in the gravity and quantum effect of black hole.

An effective method to investigate black hole is the researches of quasinormal modes (QNMs) [1–37]. A perturbation could make black hole vibrate, the frequency of vibration, i.e. Quasinormal modes frequency (QNF), is independent of the perturbation, but depend on

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the property of black hole, so QNF is inherent frequency in the spacetime. In physics, the perturbation could be scalar field perturbation which determined by Klein-Gordon equation, Dirac perturbation which determined by Dirac equation, electromagnetic perturbation which determined by Einstein-Maxwell equations and gravitational perturbation from gravitational weak field theory. These perturbation lead to different QNF, which reflect the inherent characteristic of black hole and can help people deeper understand the essence of gravity. However, it is difficulty to solve the QMNs equation analytically, so several numerical methods is proposed. WKB approach is a simple but effective method in thus numerical methods. This way is firstly proposed by Shchutz and Will [38], and then is improved by Iyer, Will [39] and Konoplya [40–44]. Under their efforts, the precision of QNF's numerical results from WKB approach is reliable.

On the other hand, recently, a non-relativistic renormalisable gravitational theory is proposed by Hořava [45], and attract considerable interest. After motivated by Lifshitz theory in solid state physics, the theory is usually referred to as the Hořava-Lifshitz (HL) theory. This theory may provide a candidate for a ultraviolet completion theory of Relativity. Physical scientists have discussed this theory widely [46–61], and several solutions in Hořava-Lifshitz theory, such as non-charged black hole, charged black hole and cosmological spacetime, are obtained. What's more, Chen, Jing and Konoplya have researched the massless scalar QNMs of deformed Hořava-Lifshitz black hole [62, 63], and then Wang and Gui study the Dirac QNMs in this spacetime [64]. Their works is helpful to discuss the property of Hořava-Lifshitz gravity. However, up to now, nobody investigate the electromagnetic field QNMs of this black hole, it is no other than the work of this paper.

The outline of this paper is as follows. We write vector field perturbation equation of Hořava-Lifshitz black hole in Sect. 2, and numerical calculate the QNF by using six-order WKB approach in Sect. 3. The Sect. 4 include some discussion and conclusion.

2 Vector Field Perturbation of Hořava-Lifshitz Black Hole

As we all know, electromagnetic field in spacetime can be described by Maxwell equations in curved spacetime, and this equations determined the dynamical behavior of massless vector field particles. In a spherical symmetrical spacetime

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1}$$

the radial electromagnetic field perturbation equation of this black hole can be written as Schrödinger-like equation

$$\frac{d^2\Phi}{dr_*^2} + (\omega^2 - V(r))\Phi = 0 \tag{2}$$

where

$$V(r) = f(r)\frac{l(l+1)}{r^2}, \tag{3}$$

and $r_* = \int \frac{dr}{f(r)}$. If spacetime (1) is asymptotically approach flat spacetime, the QNMs of the black hole is defined as ingoing mode at horizon but outgoing mode at infinity.

In the Hořava-Lifshitz theory, the deformed action with $\Lambda_W \to 0$ is given by [61]

$$S_{HL} = \int dx^3 dt \sqrt{g} N \left\{ \frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda K^2) + \frac{\kappa^2 \mu^2 (\Lambda R - 3\Lambda_W^2)}{8(1-3\lambda)} + \frac{\kappa^2 \mu^2 (1-4\lambda)}{32(1-3\lambda)} R^2 - \frac{\kappa^2}{2\omega^4}\left(C_i j - \frac{\mu\omega_c^2}{2} R_i j\right)\left(C^i j - \frac{\mu\omega_c^2}{2} R^i j\right) + \mu^4 R \right\} \tag{4}$$

where, extrinsic curvature $K_{ij} = \frac{1}{2N}(\partial_t g_{ij} - \nabla_i N_j - \nabla_j N_i)$ and Cotton tensor $C_{ij} = \epsilon^{ijk}\nabla_k (R_l^j - \frac{1}{4} R \delta_l^j)$. For $\lambda = 1 (\omega_c = 16\mu^2\kappa^{-2})$, a spherically symmetric black hole solution is obtained in Refs. [61–64]. In this spacetime,

$$f(r) = \frac{2(r^2 - 2Mr + \alpha)}{r^2 + 2\alpha + \sqrt{r^4 + 8\alpha M r}} \tag{5}$$

where $2\omega_c = \alpha^{-1}$. It is obvious that it become Schwarzschild geometry when $\alpha = 0$, but this spacetime is distinctly different from Reissner-Norström spacetime. In fact, when α is small, we can expand (5)

$$f(r) \simeq 1 - \frac{2M}{r} + \frac{4\alpha M^2}{r^4}, \tag{6}$$

which has not the term of r^{-2}. We will research the electromagnetic QNMs in this spacetime. As contrast, we also show the QNMs of Reissner-Norström black hole and a regular charged black hole [65], which components of metrics respectively are

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \quad \text{for Reissner-Norström black hole case} \tag{7}$$

and

$$f(r) = 1 - \frac{2M(1 - \tanh(Q^2/2Mr))}{r} \quad \text{for regular charged black hole case} \tag{8}$$

In Sect. 3, we will use six-order WKB approach to research the electromagnetic field QNF ω of Hořava black hole.

3 Numerical Results from Six-order WKB Approach

In six-order WKB approach which is proposed by Konoplya, the QNF ω is determined by the formula

$$\frac{i(\omega^2 - V)}{\sqrt{-2V''}}|_{r=r_0} = n + \frac{1}{2} + \Lambda_2 + \Lambda_3 + \Lambda_4 + \Lambda_5 + \Lambda_6 \tag{9}$$

where the x_0 is the maximum value of $V(r)$, and Λ_n represent the n-th order correction, and the concrete form of the corrections can be found in Refs. [38, 39] and [40]. We do the numerical calculation through Mathematica 5.2. The key point is that the computing speed may be determined by the form of initial value, i.e., the fraction initial value results faster computing speed than decimal initial value. What's more, the results may be tiny difference between fraction initial value and decimal initial value. We guess the answer for that should

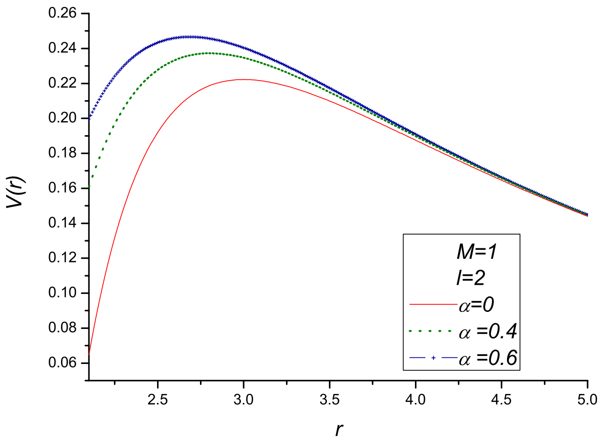

Fig. 1 The potential of electromagnetic QMNs of Hořava-Lifshitz black hole with $\alpha = 0$, $\alpha = 0.4$ and $\alpha = 0.6$ for the parameters $M = 1$ and $l = 2$

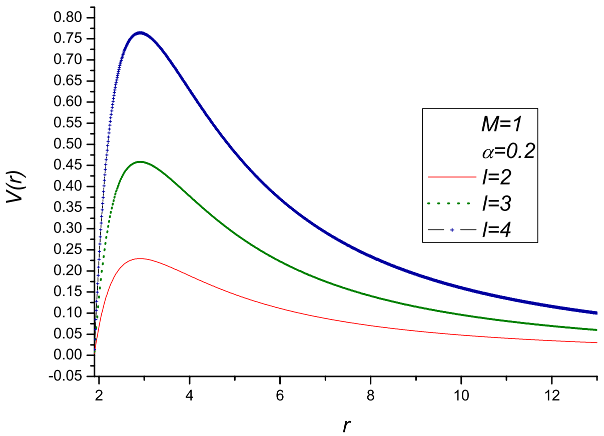

Fig. 2 The potential of electromagnetic QMNs of Hořava-Lifshitz black hole with $l = 2$, $l = 3$ and $l = 4$ for the parameters $M = 1$ and $\alpha = 0.2$

be some circulating decimal cut off in computing process. Therefore, in this paper, we use fraction initial value.

First of all, we program to compute electromagnetic QMNs of Hořava-Lifshitz black hole by using six-order WKB approach. In our work, we take $M = 1$.

In Fig. 1, we plot the potential of electromagnetic QMNs' equation with different α in Hořava-Lifshitz spacetime. It show that the peak value of potential increases but the location of peak r_0 decrease as α increase.

In Fig. 2, we plot the potential of electromagnetic QMNs' equation with different l in Hořava-Lifshitz spacetime. It show that the peak value of potential increases but the location of peak r_0 keep the same value as α increase. It is easy to understand that the r_0 are the same value, because $l(l + 1)$ is constant in the potential $V(r)$ of (3), and l ($l \neq 0$) won't have any contribution in the calculation about r_0.

From Tables 1–9, we find that the real parts of QNF increase but the absolute value of imaginary parts decrease as the α increase in Hořava-Lifshitz spacetime. However, Both the real parts and the absolute value of imaginary parts increase as the Q increase in charged spacetimes. It show that the QNMs' damping of Hořava-Lifshitz black hole is slower with larger α, but the damping is rapider with larger Q in charged spacetime.

Table 1 QNF of vector field perturbation of Hořava-Lifshitz black hole with $l = 2$ for black hole's mass $M = 1$

α	$\omega(n = 0)$	$\omega(n = 1)$	$\omega(n = 2)$
0	$0.45759 - 0.09501i$	$0.43653 - 0.29073i$	$0.40091 - 0.50173i$
0.2	$0.46591 - 0.09187i$	$0.44721 - 0.28052i$	$0.41539 - 0.48215i$
0.4	$0.47518 - 0.08819i$	$0.45845 - 0.26863i$	$0.42957 - 0.45969i$
0.6	$0.48575 - 0.08364i$	$0.47044 - 0.25390i$	$0.44308 - 0.43197i$

Table 2 QNF of vector field perturbation of Reissner-Nordström black hole with $l = 2$ for black hole's mass $M = 1$

Q	$\omega(n = 0)$	$\omega(n = 1)$	$\omega(n = 2)$
0	$0.45759 - 0.09501i$	$0.43653 - 0.29073i$	$0.40091 - 0.50173i$
0.2	$0.46083 - 0.09523i$	$0.43993 - 0.29134i$	$0.40459 - 0.50260i$
0.4	$0.47115 - 0.09586i$	$0.45082 - 0.29308i$	$0.41643 - 0.50496i$
0.6	$0.49090 - 0.09672i$	$0.47175 - 0.29530i$	$0.43937 - 0.50755i$

Table 3 QNF of vector field perturbation of charged regular black hole with $l = 2$ for black hole's mass $M = 1$

Q	$\omega(n = 0)$	$\omega(n = 1)$	$\omega(n = 2)$
0	$0.45759 - 0.09501i$	$0.43653 - 0.29073i$	$0.40091 - 0.50173i$
0.2	$0.46083 - 0.09523i$	$0.43993 - 0.29134i$	$0.40459 - 0.50260i$
0.4	$0.47115 - 0.09586i$	$0.45081 - 0.29308i$	$0.41642 - 0.50497i$
0.6	$0.49084 - 0.09675i$	$0.47167 - 0.29538i$	$0.43927 - 0.50772i$

Table 4 QNF of vector field perturbation of Hořava-Lifshitz black hole with $l = 3$ for black hole's mass $M = 1$

α	$\omega(n = 0)$	$\omega(n = 1)$	$\omega(n = 2)$
0	$0.65690 - 0.09562i$	$0.64174 - 0.28973i$	$0.61379 - 0.49206i$
0.2	$0.66796 - 0.09247i$	$0.65442 - 0.27985i$	$0.62937 - 0.47415i$
0.4	$0.68037 - 0.08877i$	$0.66826 - 0.26825i$	$0.64565 - 0.45325i$
0.6	$0.69457 - 0.08421i$	$0.68349 - 0.25403i$	$0.66245 - 0.42777i$

4 Conclusion

In this paper, we investigate the electromagnetic field QNMs of Hořava-Lifshitz black hole, and our results show that the QNMs' damping of Hořava-Lifshitz black hole is slower with α increase. This property could become a criterion differentiating Hořava-Lifshitz gravity from other gravity.

On the other hand, it is well known that larger massive black hole could radiate massless particle because the Hawking temperature is low, but tiny black holes with higher Hawking temperature could radiate massive particles. Recently, experiments of LHC may create

Table 5 QNF of vector field perturbation of Reissner-Nordström black hole with $l = 3$ for black hole's mass $M = 1$

Q	$\omega(n=0)$	$\omega(n=1)$	$\omega(n=2)$
0	$0.65690 - 0.09562i$	$0.64174 - 0.28973i$	$0.61379 - 0.49206i$
0.2	$0.66144 - 0.09583i$	$0.64640 - 0.29036i$	$0.61867 - 0.49302i$
0.4	$0.67593 - 0.09645i$	$0.66129 - 0.29212i$	$0.63430 - 0.49569i$
0.6	$0.70360 - 0.09727i$	$0.68980 - 0.29442i$	$0.66438 - 0.49895i$

Table 6 QNF of vector field perturbation of charged regular black hole with $l = 3$ for black hole's mass $M = 1$

Q	$\omega(n=0)$	$\omega(n=1)$	$\omega(n=2)$
0	$0.65690 - 0.09562i$	$0.64174 - 0.28973i$	$0.61379 - 0.49206i$
0.2	$0.66144 - 0.09583i$	$0.64640 - 0.29036i$	$0.61867 - 0.49302i$
0.4	$0.67592 - 0.09645i$	$0.66128 - 0.29212i$	$0.63429 - 0.49570i$
0.6	$0.70351 - 0.09730i$	$0.68971 - 0.29450i$	$0.66427 - 0.49910i$

Table 7 QNF of vector field perturbation of Hořava-Lifshitz black hole with $l = 4$ for black hole's mass $M = 1$

α	$\omega(n=0)$	$\omega(n=1)$	$\omega(n=2)$
0	$0.85310 - 0.09586i$	$0.84127 - 0.28932i$	$0.81872 - 0.48784i$
0.2	$0.86703 - 0.09273i$	$0.85645 - 0.27966i$	$0.83622 - 0.47083i$
0.4	$0.88270 - 0.08903i$	$0.87322 - 0.26826i$	$0.85500 - 0.45085i$
0.6	$0.90065 - 0.08449i$	$0.89197 - 0.25428i$	$0.87512 - 0.42644i$

Table 8 QNF of vector field perturbation of Reissner-Nordström black hole with $l = 4$ for black hole's mass $M = 1$

Q	$\omega(n=0)$	$\omega(n=1)$	$\omega(n=2)$
0	$0.85310 - 0.09586i$	$0.84127 - 0.28932i$	$0.81872 - 0.48784i$
0.2	$0.85894 - 0.09607i$	$0.84721 - 0.28994i$	$0.82484 - 0.48883i$
0.4	$0.87759 - 0.09668i$	$0.86616 - 0.29172i$	$0.84439 - 0.49163i$
0.6	$0.91318 - 0.09750i$	$0.90241 - 0.29406i$	$0.88190 - 0.49518i$

tiny black holes, and it is a chance to test modern gravitational theory. Therefore, the researches of massive vector field QNMs also is significant. However, Ref. [66] tell us that the higher-order WKB approach fail to apply in massive vector field case. What more, it is difficulty to get the massive vector field Schrödinger-like equation with $l \neq 0$. Nevertheless, we will focus on massive QNMs of Hořava-Lifshitz black hole in the future. Meanwhile, Horava-Lifshitz theory has scalar, non-scalar curvature, and coordinate singularities. How these singularities impact on QNMs' characteristic need to be further investigated.

Table 9 QNF of vector field perturbation of charged regular black hole with $l = 4$ for black hole's mass $M = 1$

Q	$\omega(n=0)$	$\omega(n=1)$	$\omega(n=2)$
0	$0.85310 - 0.09586i$	$0.84127 - 0.28932i$	$0.81872 - 0.48784i$
0.2	$0.85894 - 0.09607i$	$0.84721 - 0.28994i$	$0.82484 - 0.48883i$
0.4	$0.87758 - 0.09668i$	$0.86616 - 0.29172i$	$0.84438 - 0.49164i$
0.6	$0.91307 - 0.09752i$	$0.90230 - 0.29414i$	$0.88177 - 0.49532i$

Acknowledgements This work is supported by National Natural Science Foundation of China No. 10773008 and 11075224, Chongqing University Postgraduates Science and Innovation Fund, Project Number 200811B1A0100299.

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